Bootstrap Methods for Inference in a SUR model with Autocorrelated Disturbances

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Abstract

Although the Parks (1967) estimator for a SUR model with AR disturbances is efficient both asymptotically and in small samples, Kmenta and Gilbert (1970) and more recently Beck and Katz (1995) note that estimated standard errors tend to be biased downward as compared with the true variability of the estimates. This bias leads to tests that show over-rejection and to confidence intervals that are too small. We suggest bootstrapping the tests to correct this inference problem. After illustrating the over rejection associated with the estimated asymptotic standard errors, we develop a bootstrap approach to inference for this model, illustrate its use, and show using Monte Carlo methods that the bootstrap gives rejection probabilities close to the nominal level chosen by the researcher.

1. Introduction:

This paper presents bootstrap methods for inference in a seemingly unrelated regression (SUR) model with autocorrelated disturbances. We show via a Monte Carlo study that bootstrap methods are capable of correcting and largely eliminating the level-distortion that occurs with the use of the estimated covariance matrix associated with the Parks estimator.

The Parks estimator (1967) was designed as an efficient estimator for systems of equations with both serially and contemporaneously correlated disturbances. Such models include the SUR model and various restricted forms of it such as pooled time series cross-section models. In this context, the Parks estimator was shown to be consistent and asymptotically more efficient than competing unbiased estimators, including the Zellner (1962) estimator, which corrects for contemporaneous correlation but not for serial correlation, and ordinary least squares (OLS), which corrects for neither.[1] Since time series cross section data in social science research often fits this framework, the Parks estimator has

[1] The unbiasedness property was shown only later. Kakwani's (1967) argument for the Zellner estimator was extended by Magnus (1978) to cover this estimator. Andrews (1987) provides a comprehensive treatment.

been widely used and is available in many econometric software packages including RATS, SHAZAM, SAS, Stata, and Eviews.

Following Parks (1967), a series of Monte Carlo studies demonstrated a small sample efficiency gain from the use of the Parks estimator, although these studies were limited in their consideration of models with relatively few equations (the cross-section dimension, M) and with substantially more time series observations, T, than equations. i.e. with T>>M. Kmenta and Gilbert (1970) explores a model with M=2 equations and T=10, 20, and 100 observations, together with several covariance specifications. Their results confirm, for all sample sizes, an improvement in efficiency associated with the Parks estimator, even in cases without cross equation correlation.[2] Kmenta and Gilbert (1968) provide Monte Carlo results for the Zellner, SUR model; Zellner (1963) provides some exact finite sample results in relatively simple cases. Guilkey and Schmidt (1973) reaffirm these findings for the SUR model with a more general, vector autoregressive error process and provide the basis for an improved treatment of the first observation that preserves stationarity. Maeshiro (1980) provides evidence on the problems created by trended exogenous variables, and the importance in these cases of retaining the initial observation. Doran and Griffiths (1983) provide additional evidence on this situation and implement the Guilkey and Schmidt (1973) stationarity result in their estimation procedure.

While the small sample efficiency of the Parks estimator is a desirable property, most inference depends on having reasonable estimates of standard errors or more generally the covariance matrix. Although the estimated covariance matrix provided by Parks (1967) is consistent, Kmenta and Gilbert (1970) note in their Monte Carlo study that the estimated standard errors of the Parks estimator appear to be biased when compared with the true variability of the estimates, even in samples as large as 100. More recently, Beck and Katz (1995) showed that the estimated standard errors for the Parks estimator have severe downward bias with pooled cross-section, time series data where the time dimension T is small relative to the number of cross-sections M. They also show that the actual coverage probabilities for confidence intervals could be well below their nominal levels.

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[2] They considered the relative small sample efficiency of OLS, Zellner, and several variants of the Parks estimator, all of which involve dropping the initial observation.

For testing hypotheses in a SUR model with autocorrelated errors, Beck and Katz (1995) recommend using the inefficient OLS estimator together with standard errors from the corresponding "sandwich" covariance matrix that is appropriate given the assumed structure for the disturbances. We think, however, that this is not the best advice for it ignores a bootstrap approach that preserves the use of the efficient estimator while eliminating the size or level-distortion associated with the biased standard errors or covariances.

It is clearly important for researchers working with time series cross-section data to be aware of the Beck and Katz (1995) findings regarding the potential downward bias and over-rejection when using the estimated standard errors for the Parks estimator. Although their method reduces this level-distortion, it generally fails to eliminate it; it is based on an inefficient estimator, and it is difficult to implement in situations involving both contemporaneously and serially correlated disturbances.[3]

This paper shows that bootstrapped hypothesis tests constructed with the Parks estimator permits the use an efficient estimator while largely eliminating level-distortion. We illustrate the bootstrap procedures and construct a set of Monte Carlo studies using the familiar two-equation GE-Westinghouse data set from Grunfeld that has been shown to have both contemporaneous and serial correlation in the disturbances (cf. Greene 2003).

The remainder of the paper is organized as follows. In Section 2 we present the model and describe the estimation procedures that show the best performance. In Section 3 we discuss the level-distortion that arises when the estimated covariance or standard errors are used. We present results of a Monte Carlo study extending Kmenta and Gilbert (1970) and Beck and Katz (1995) and demonstrate the level-distortion involved in hypothesis tests based on the Parks estimator. Section 4 reviews the key results of bootstrap theory as they apply to this context and describes the procedures involved in performing both parametric and non-parametric bootstrap tests. In Section 5 we illustrate the bootstrap and show how different the asymptotic and bootstrap critical values are for tests constructed with Grunfeld's GE-Westinghouse data. We then present the results of Monte Carlo experiments showing that the

[3] By assuming that "Any serial correlation of the errors must be eliminated before the panel-corrected standard errors are calculated." Beck and Katz (1995) effectively restrict their analysis to the Zellner case involving only contemporaneously correlated disturbances. Unfortunately the two parts of the problem, contemporaneous and serial correlation, are not separable in a simple way as they suggest.

bootstrap essentially eliminates the level-distortion for tests based on the Parks estimator and its estimated standard errors or covariance. Section 6 presents our conclusions.

2. Specification and Estimation of the SUR Model with Autocorrelated Errors

The Parks estimator was designed as an efficient estimator for the following model, written by equation. Let

$$y_i = X_i\beta_i + \varepsilon_i \quad i = 1,\ldots,M, \tag{1}$$

where y_i and ε_i are $T \times 1$ vectors, X_i is $T \times k_i$, and β_i is $k_i \times 1$. We can then further compress the notion by stacking the M equations in the compact form

$$y = X\beta + \varepsilon \tag{2}$$

where $y = \begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix}$, $X = \begin{pmatrix} X_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & X_M \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_M \end{pmatrix}$, and $\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_M \end{pmatrix}$, and with

$E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \Omega$.

The specification of the covariance structure is simplified by arranging the data by observation, t, rather than by equation. Then it is assumed that the disturbance vector, $\varepsilon_{(t)} = (\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Mt})'$ is generated by a stationary, first-order autoregressive process

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{Mt} \end{pmatrix} = \begin{pmatrix} \rho_1 & 0 & \cdots & 0 \\ 0 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_M \end{pmatrix} \begin{pmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \\ \vdots \\ \varepsilon_{Mt-1} \end{pmatrix} + \begin{pmatrix} v_{it} \\ v_{2t} \\ \vdots \\ v_{Mt} \end{pmatrix} \tag{3}$$

or in matrix notation $\varepsilon_{(t)} = R\varepsilon_{(t-1)} + v_{(t)}$,[4] where the $v_{(t)}$ are independent and identically distributed random variables with $E(v_{(t)}) = 0$ and covariance matrix

$$E(v_{(t)}v_{(t)}') = \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \cdots & \sigma_{MM} \end{pmatrix} \tag{4}$$

The diagonal structure of the *R* matrix implies that each equation or cross-section unit exhibits its own serial correlation coefficient, and the innovations $v_{(t)}$ are contemporaneously correlated with covariance matrix Σ.[5]

The most general model that we will consider involves the diagonal *R* matrix, with *M* parameters, specifying the serial correlation together with a full, symmetric Σ matrix, with $M(M+1)/2$ parameters, specifying the contemporaneous covariance.

If Ω is known, the generalized least squares estimator for the coefficients in this model is

$$\hat{\beta} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y \tag{5}$$

The inverse of Ω, which is $MT \times MT$, can be difficult to compute when *M* and *T* are large. In addition, in most applications, Ω is not known and has to be estimated. The Parks estimator addresses this problem by transforming the data to remove the serial correlation then applying the SUR estimator. Transformation of observation 2,…,*T* involves the familiar weighted first differencing. Transformation of the first observation is more complicated and involves parameters from both R and Σ.[6] There are a variety of ways to implement the estimator. In

[4] To clarify notation, notice that the vector ε_i contains the T disturbances for the *ith* equation whereas the vector $\varepsilon_{(t)}$ contains the *M* disturbances for different equations or cross-sectional elements at time t.

[5] Judge et. al (1985) provides a useful discussion of both the covariance structure and estimation methods.

[6] Although the estimation procedure proposed by Parks (1967) preserved the initial observation, it did not preserve stationarity. Guilkey and Schmidt (1973) provided the theoretical details to correct that problem. Doran and Griffiths (1983) implemented the modified procedure, which is now described well in Judge et al (1985). The procedures we discuss here incorporate that correction.

steps 1-5 below, we describe the feasible GLS procedure that appears consistently to show the best performance.[7]

1. Compute the SUR estimates and residuals
2. Use these residuals to estimate the serial correlation coefficients for each equation
3. Transform the data as ρ_i-weighted first differences (*T-1* observations), run OLS on the transformed data, and use E, the resulting matrix of residuals to compute $\hat{\Sigma} = \left(\frac{1}{T}\right) E'E$
4. construct the transformation matrix, $\hat{P}$, such that:

$$\hat{P}\hat{\Omega}\hat{P}' = \hat{\Sigma} \otimes I \tag{6}$$

This transformation leads to a manageable form of the inverse of the system covariance matrix,

$$\hat{\Omega}^{-1} = \hat{P}'(\hat{\Sigma}^{-1} \otimes I)\hat{P} \tag{7}$$

The Parks FGLS estimator is then:

5. $\hat{\beta}_P = \left(X'\hat{P}'(\hat{\Sigma}^{-1} \otimes I)\hat{P}X\right)^{-1} \left(X'\hat{P}'(\hat{\Sigma}^{-1} \otimes I)\hat{P}y\right)$ (8)

or

$$\hat{\beta}_P = \left(X^{*'}(\hat{\Sigma}^{-1} \otimes I)X^*\right)^{-1} \left(X^{*'}(\hat{\Sigma}^{-1} \otimes I)y^*\right) \tag{9}$$

where $X^* = \hat{P}X$ and $y^* = \hat{P}y$.

A consistent estimator for the covariance matrix of the Parks estimator is:

$$\hat{V}(\hat{\beta}_P) = \left(X'\hat{P}'(\hat{\Sigma}^{-1} \otimes I)\hat{P}X\right)^{-1} = \left(X^{*'}(\hat{\Sigma}^{-1} \otimes I)X^*\right)^{-1} \tag{10}$$

Details of the transformation matrix *P* are given in Judge (1985). Briefly, for observations 2,...,*T*, X^* and y^* represent the familiar weighted first differences using the estimated ρ_is. For the first observation, the transformation that preserves stationarity involves a complicated mixture of the parameters from both the *R* and Σ matrices.

[7] Messemer (2003) derives the maximum likelihood estimator. It seems to dominate the FGLS estimators in terms of small sample efficiency, but is more difficult to compute.

3. Level-distortion in Hypothesis Tests

3.1 Level-distortion.

Hypothesis tests using the estimate of the asymptotic covariance matrix (10) rely on the asymptotic distribution of the test statistic, usually Normal or Chi-squared, to compute *p*-values or to derive critical values with nominal level of significance α. A test shows level-distortion if the true probability of observing test statistics above a given critical value under the null hypothesis departs from the nominal level.

In this section we demonstrate the level-distortion for a variety of tests based on the Parks estimator and its estimated asymptotic covariance with a Monte Carlo study using the classic General Electric and Westinghouse data from Grunfeld (1958), which has been widely used to illustrate the SUR model.

3.2 Model Estimates.

Grunfeld sought to explain a firm's net investment in terms of its expected profits and its desired capital stock, using data from 1935-1954. For two firms, General Electric and Westinghouse, his model involves a system of equations with M=2, T=20, and k_1=k_2=3 (including the constant). Statistical tests suggest that disturbances show both contemporaneous and serial correlation, so that the Parks estimator is appropriate. Table 1 shows the Parks estimates for the two regressions and their estimated asymptotic standard errors. The last column shows the estimated autoregressive coefficients. These can be compared with estimates based on OLS and SUR shown in Theil (1971, chapter 7).

Table 1. Parks Estimates of the GE-Westinghouse Model with Asymptotic Standard Errors

	Constant	Profit	Capital	AR(1)
GE parameter estimates	-29.40 (28.66)*	0.043 (0.013)	0.119 (0.036)	0.50
Westinghouse parameter estimates	2.63 (7.67)	0.051 (0.051)	0.067 (0.058)	0.29

* Estimated asymptotic standard errors in parentheses

3.3 Hypothesis Tests.

To demonstrate the level-distortion of hypothesis tests constructed using the Parks estimated coefficients and their asymptotic standard errors, we construct Wald test statistics. For null hypotheses of the form $R\beta = r$, the Wald test statistics are constructed as:

$$g = \left(R\hat{\beta} - r\right)' \left(R\hat{V}(\hat{\beta})R'\right)^{-1} \left(R\hat{\beta} - r\right) \tag{11}$$

where $\hat{V}(\hat{\beta})$ is calculated from (10) and where the restriction matrix R has q rows (the number of restrictions). The test statistic, g, is asymptotically distributed χ_q^2. In the following Monte Carlo studies, we examine three tests. First, we test for the significance of the first independent variable in the GE estimating equation. Second, we test a single cross-equation restriction that the coefficient on the first independent variable is the same for both GE and Westinghouse. Finally, we test a joint cross-equation hypothesis that the coefficients on both independent variables are the same for both firms. The first and second test statistics are distributed χ_1^2, and the third, χ_2^2.

3.3 Monte Carlo study of level-distortion with tests based on asymptotic standard errors of the Parks estimator

A Monte Carlo study of a test's level- performance involves the following steps:

1. Choose parameters for the model satisfying the null hypothesis
2. Generate a sample data set.
3. Using this data set, test the null hypothesis using critical values from the χ_q^2 distribution with a nominal level of significance, α.
4. Repeat steps 2 and 3 to get *N* Monte Carlo samples, test statistics, and decisions.
5. Compare the frequency of rejection in the Monte Carlo samples to the nominal level of the test.

As the data generating process, we use Grunfeld's dataset with the Parks estimates for AR(1) parameters and contemporaneous covariance matrix. The regression parameters are modified separately for each of the three tests so as to satisfy the relevant null hypothesis. For each of the 1000 Monte Carlo replications, we calculate the Parks estimates and compute the test statistic. We use a 5 percent nominal level for all of the tests. The actual level for each test, as estimated from the Monte Carlo experiment, is the number of rejections divided by the number of replications. The results of the Monte Carlo experiments are shown in Table 2 below.

Table 2: Rejection Probabilities for Tests Based on the Parks Estimator and Its Estimated Asymptotic Covariance

Test-Statistic	MC-estimated Level	Nominal Level
g_1	0.165	0.05
g_2	0.126	0.05
g_3	0.204	0.05

These results show significant distortion for all three tests, with consistent over-rejection as compared with the nominal levels. Because of this problem Beck and Katz

(1995) urged abandoning the Parks estimator altogether in favor of an estimator which has lower level-distortion. As an alternative they suggested using OLS estimates together with what they call panel corrected standard errors, i.e. standard errors taken from an estimate of the OLS covariance, which in the present context has the form

$$\hat{V}(\beta_{OLS}) = (X'X)^{-1}(X'\hat{\Omega}X)(X'X)^{-1}, \tag{12}$$

Tests using their approach also suffer from level-distortion, however, and although the distortion may be smaller than with the Parks estimator, it remains of potential importance. Furthermore, we are not forced to choose between efficiency and level-distortion. With modern bootstrap techniques, we can use a test statistic based on the efficient Parks estimator and then correct for the level-distortion.

4. Bootstrap Inference in a SUR Model with Autocorrelated Disturbances

The bootstrap has been shown to improve on asymptotic approximations to the distribution of test statistics when the statistics are asymptotically pivotal. A statistic is asymptotically pivotal when its asymptotic distribution does not depend on nuisance parameters. Statistics such as Wald and Student-t that would be natural choices in the present context have this property.[8]

In this section, we draw upon and extend the bootstrap literature by describing both parametric and non-parametric bootstrap methods for the SUR model with autocorrelated errors.[9] Horowitz (1997) and others have provided extensive surveys of the bootstrap literature. Horowitz, p.201, gives a succinct statement of the key bootstrap results:

> "The bootstrap provides a higher-order asymptotic approximation to critical values for tests based on "smooth" asymptotically pivotal statistics. When a bootstrap-based critical value is used for such a test, the difference between the test's true and nominal levels decreases more rapidly with increasing sample size than it does when the critical value is obtained from first-order asymptotic theory. Given a sufficiently large sample, the nominal level of the test will be closer to the true level when a bootstrap critical value is used than when a critical value based on first-order asymptotic theory is used."

[8] Early bootstrapping papers focused on correcting biased standard errors, but subsequent theoretical work showed that the proper focus is the test statistics themselves. Horowitz (1997) reviews the literature which explains that while bootstrapping can be applied to statistics that are not asymptotically pivotal such as regression coefficients, it does not provide higher-order approximations to their distributions.
[9] Rilstone and Veall (1996) discuss bootstrapping for the standard SUR model.

4.1 *Parametric Bootstrap for the SUR Model with AR(1) Disturbances*

The simplest type of bootstrap is a parametric bootstrap, but it requires a more complete specification of the data generating process including a specific assumption about the distribution of the disturbances. Below we give the steps for implementing a parametric bootstrap test of a null hypothesis, H_0, where we assume Normality of the disturbances.

1. Estimate $\hat{\beta}$ from the unrestricted model and compute the test statistic, g above. Call this test statistic $\hat{g}$. Re-estimate the model under the restrictions imposed by the null hypothesis to obtain $\tilde{\beta}$, $\tilde{R}$, and $\tilde{\Sigma}$.
2. Generate a bootstrap sample satisfying the null hypothesis using the restricted estimates $\tilde{\beta}$, $\tilde{R}$, and $\tilde{\Sigma}$ as the parameters of the data generating process and drawing a random sample of disturbances from the assumed distributiontion. In the present context, the moderately complicated generation of the disturbances can be described as follows:
    a. Draw an $M \times T$ matrix $U = \{u_{(t)}\}$ of standard Normal random variables.
    b. Transform the columns of U to have covariance Σ, i.e. construct $V = \{v_{(t)}\} = \{H'u_{(t)}\} = H'U$ where H' is a lower triangular Cholesky factor of Σ such that $H'H = \Sigma$.
    c. Construct the disturbance vectors $\varepsilon_{(t)}$. The matrix A needed to transform the first observation's disturbance is defined as $A = H'(B')^{-1}$ where B' is the lower triangular Cholesky factor of $V_0 = E(\varepsilon_{(t)}\varepsilon'_{(t)})$ such that $B'B = V_0$ and $V_0 = \left\{\dfrac{\sigma_{ij}}{1-\rho_i\rho_j}\right\}$. If the $M \times T$ matrix $E = \{\varepsilon_{(t)}\}$, then it can be constructed as follows: $\varepsilon_{(1)} = Av_{(1)}$ and $\varepsilon_{(t)} = R\varepsilon_{(t-1)} + v_{(t)}$ for $t = 2,...,T$.
3. Estimate parameters for the unconstrained model from the first bootstrap sample (b=1), and compute the test statistic for the bootstrap sample.
4. Repeat the process of generating a bootstrap sample, estimating the model, and computing the test statistic until we have *B* bootstrap samples and test statistics $g_1, g_2, ..., g_B$. Davidson and MacKinnon (2004) recommend choosing *B* such that $\alpha(B+1)$ is integer

where α is the level of significance of the test, e.g. B=999. Estimate the α level critical value for the test, $g_{c\alpha}$, as the (1-α)th quantile from the empirical distribution of the g_bs.

5. Reject H_0 at nominal α level if $\hat{g} > g_{c\alpha}$; alternatively compute a p-value in step 4 as the fraction of the bootstrap samples with $g_b > \hat{g}$.

The above procedure can be suitably modified at steps 1 and 3 to deal with test statistics that depend on estimates of the restricted model, i.e. Lagrange multiplier tests or on estimates of both restricted and unrestricted models, i.e. likelihood ratio tests.

4.2 *Non-Parametric (or semi-parametric) Bootstrap:*

A non-parametric bootstrap follows the same general outline as that given above but instead of using a parametric specification of the distribution of the disturbances it uses re-sampling with replacement from the original residuals , which are used as an empirical representation of the disturbance distribution. In the present context, the process is complicated by the serial correlation, but the following process provides a feasible approach.

1'. Estimate the parameters of the unrestricted model and compute the test statistic, g above. Call this test statistic $\hat{g}$. Re-estimate the model under the restrictions imposed by the null hypothesis to obtain $\tilde{\beta}$, $\tilde{R}$, $\tilde{\Sigma}$, and the $T \times M$ matrix of residuals $\tilde{E}$.

2'. Reverse the steps in 2 above to get estimates of the "original" untransformed $u_{(t)}$s. Let $\tilde{v}_{(t)} = \tilde{\varepsilon}_{(t)} - \tilde{R}\tilde{\varepsilon}_{(t-1)}$ for $t = 2,...,T$ and $\tilde{v}_{(1)} = A^{-1}\tilde{\varepsilon}_{(1)}$. Then let $\tilde{u}_{(t)} = (H')^{-1} v_{(t)}$. The $\tilde{u}_{(t)}$s are the empirical representatives of the distribution of these disturbances. Draw a sample of T $\tilde{u}_{(t)}$ vectors with replacement from this empirical distribution to form the columns of U. Then proceed as above in the remaining steps 3 through 5.

5. An Illustration and Some Monte Carlo Results for Bootstrap Procedures

Below we illustrate the parametric bootstrap using the Grunfeld GE-Westinghouse data. We consider the three test statistics described above based on the Parks estimates shown in Table 1. Table 3 presents the calculated test statistics, along with their asymptotic critical values calculated from the relevant χ^2 distribution and the parametric bootstrap critical values calculated as in steps 1-4 in section 4.

Table 3: Test Statistics along with Asymptotic and Bootstrap Critical Values

Test Statistic	Asymptotic Critical Value	Bootstrap Critical Value
g_1=11.19	3.84	8.31
g_2=0.46	3.84	6.86
g_3=1.20	5.99	11.27

As expected, the bootstrap critical values are above their asymptotic counterparts. Thus they will tend to correct the over-rejection of the Parks estimator found here and in the literature. Although in this instance the decision to reject or fail to reject is unaltered, it is apparent that the critical values differ significantly, and could affect the test decision in some instances.

Through Monte Carlo experiments, Rilstone and Veall (1996) demonstrate that the bootstrap largely eliminates level-distortion for tests in the SUR model. We show, through Monte Carlo experiments described below that the bootstrap largely eliminates the level-distortion for tests in the SUR model with autocorrelated errors. The results in table 4 are from Monte Carlo experiments of 1000 simulations each. As in section 3, the underlying data generating process for the dependent variable uses the Parks-estimated AR(1), and contemporaneous covariance parameters, along with Grunfeld's independent variables. The regression coefficients were modified to satisfy the null hypotheses. After generating a simulated dataset (drawing from the Normal distribution), we computed the Parks estimates of the unrestricted model and the appropriate test statistic. To complete the test based upon the sample data, we obtained parametric and non-parametric bootstrap critical values as described above in section 4. For each simulation, we recorded the decision of the test. The bootstrap level reported in Table 4 below was calculated as the number of rejections of the null hypothesis based on the bootstrap critical values divided by the number of simulations. The asymptotic level from Table 2 is included for comparison.

Table 4: Rejection Probabilities for Tests Based on Bootstrap and Asymptotic Critical Values

Test-Statistic	Bootstrap Level		Asymptotic Level	Nominal Level
	Parametric	Non Parametric		
g_1	0.055	0.050	0.166	0.05
g_2	0.063	0.063	0.147	0.05
g_3	0.055	0.053	0.228	0.05

As Table 4 demonstrates, the sizable level-distortion of the Parks estimator is virtually eliminated when we use bootstrap critical values for inference on test statistics. This is true for a variety of hypothesis test statistics and for a relatively small sample size. Thus, as in the bootstrap literature for other models, we conclude that the bootstrap removes the need to choose an estimator based upon level-distortion.

6. Summary

The Parks estimator is asymptotically efficient for SUR models with autocorrelated errors, and several Monte Carlo experiments have shown it to be more efficient than OLS and SUR estimates in finite samples. Kmenta and Gilbert (1970) and more recently Beck and Katz (1995) have shown, however, that the estimated asymptotic standard errors for these estimators show significant downward bias. This bias can distort conclusions from hypothesis tests or confidence intervals based on these standard errors. In this paper we document the distortion by showing that the rejection probabilities or level of tests that rely on asymptotic critical values can be far from their nominal levels. The bootstrap, however, provides a reasonable way to rescue the situation. Bootstrap tests in this context, both parametric and non-parametric, show rejection probabilities that are close to the nominal levels. We illustrate these results using the familiar Grunfeld GE and Westinghouse data, first showing that tests based on the Parks estimator and estimates of its asymptotic covariance lead to over-rejection of the null hypothesis when the tests are based on asymptotic critical values. We then describe the procedures for both parametric and non-parametric bootstrap tests in this context and illustrate their use. Finally, based on Monte Carlo experiments, we show that the bootstrap largely eliminates level-distortion in tests based on the Parks estimator.

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